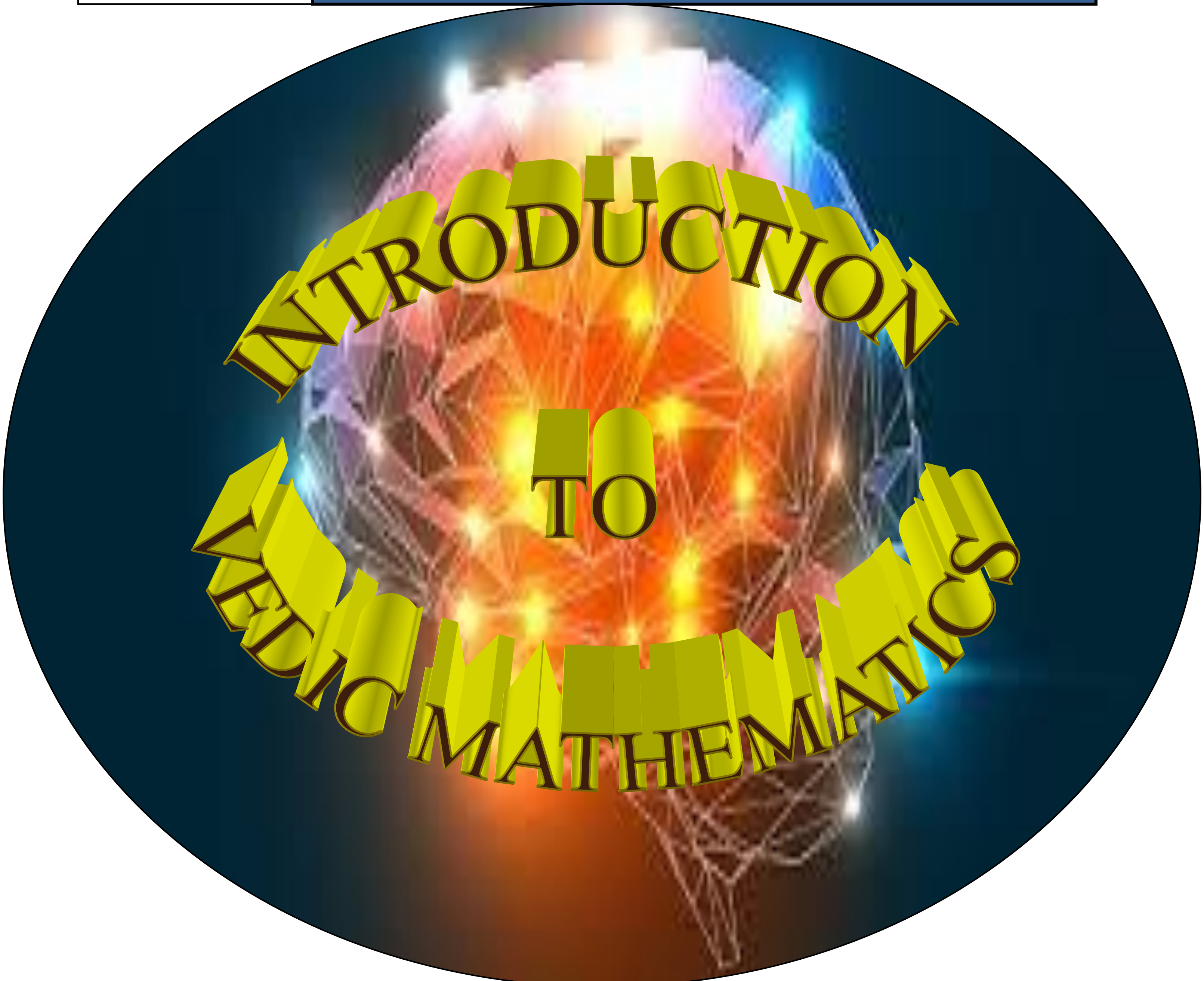
INDIAN SCHOOL SOHAR

INTRODUCTION TO VEDIC MATHEMATICS



HOLIDAY HOME WORK FOR CLASS VI. VII AND VIII

BY
DEPARTMENT OF MATHEMATICS

ASSIGNMENT – INDEX

Sixteen Sutras and their corollaries

Sl. No	Sutras	Sub sutras or Corollaries
1.	Ekādhikena Pūrvena (also a corollary)	Ānurūpyena
2.	Nikhilam Navataścaramam Daśatah	Śisyate Śesamjnah
3.	Ūrdhva - tiryagbhyām	Ādyamādyenantyamantyena
4.	Parāvartya Yojayet	Kevalaih Saptakam Gunỹat
5.	Sūnyam Samyasamuccaye	Vestanam
6.	(Ānurūpye) Śūnyamanyat	Yāvadūnam Tāvadūnam
7.	Sankalana - vyavakalanābhyām	Yāvadūnam Tāvadūnikrtya Vargañca Yojayet
8.	Puranāpuranābhyām	Antyayordasake' pi
9.	Calanā kalanābhyām	Antyayoreva
10.	Yāvadūnam	Samuccayagunitah
11.	Vyastisamastih	Lopanasthāpanabhyām
12.	Śesānyankena Caramena	Vilokanam
13.	Sopantyadvayamantyam	Gunitasamuccayah Samuccayagunitah
14.	Ekanyūnena Pūrvena	
15.	Gunitasamuccayah	
16.	Gunakasamuccayah	

SUTRA 1 - Ekadhikena Purvena

The Sutra (formula) **Ekādhikena Pūrvena** means: "By one more than the previous one".

i) Squares of numbers ending in 5 :

Now we relate the sutra to the 'squaring of numbers ending in 5'. Consider the **Example 25^2**.

Here the number is 25. We have to find out the square of the number.

For the number 25, the last digit is 5 and the 'previous' digit is 2. Hence, 'one more than the previous one', that is, 2+1=3.

The Sutra, in this context, gives the procedure'to multiply the previous digit 2 by one more than itself, that is, by 3' It becomes the L.H.S (left hand side) of the result, that is, 2 X 3 = 6. The R.H.S (right hand side) of the result is52, that is, 25.

Thus 252 = 2 X 3 / 25 = 625. In the same way,

352= 3 X (3+1) /25 = 3 X 4/ 25 = 1225;

652= 6 X 7 / 25 = 4225;

1052= 10 X 11/25 = 11025;

1352= 13 X 14/25 = 18225;

CLASS V1 - Exercise 1

Apply the formula to find the squares of the numbers

15, 45, 85, 125, 175 and verify the answers by calculation method

Example : $165^2 = (1 . 10_2 + 6 . 10 + 5)^2$.

It is of the form $(ax^2 + bx + c)^2$ for a = 1, b = 6, c = 5 and x = 10. It gives the answer P(P+1) / 25, where P = 10a + b = 10 X 1 + 6 = 16, the 'previous'. The answer is 16 (16+1) / 25 = 16 X 17 / 25 = 27225.

CLASS VII , VIII - Exercise 2

Apply Ekadhikena purvena to find the squares of the numbers

95, 225,375, 635, 745, 915, 1105, 2545.

Vulgar fractions whose denominators are numbers ending in NINE

We now take examples of 1 / a9, where a = 1, 2, -----, 9. In the conversion of such vulgar fractions into recurring decimals, Ekadhika process can be effectively used both in division and multiplication.

a) Multiplication Method: Value of 1 / 19

First we recognize the last digit of the denominator of the type 1 / a9. Here the last digit is 9.

For a fraction of the form in whose denominator 9 is the last digit, we take the case of 1 / 19 as follows:

For 1 / 19, 'previous' of 19 is 1. And one more than of it is 1 + 1 = 2.

Therefore 2 is the multiplier for the conversion. We write the last digit in the numerator as 1 and follow the steps leftwards.

Step. 1 : 1

Step. 2 : 21(multiply 1 by 2, put to left)

Step. 3 : 421(multiply 2 by 2, put to left)

Step. 4 : 8421(multiply 4 by 2, put to left)

Step. 5 : 168421 (multiply 8 by 2 =16, 1 carried over, 6 put to left)

Step. 6 : 1368421 (6 X 2 =12,+1 [carry over] = 13, 1 carried over, 3 put to left)

Step. 7 : 7368421 (3 X 2, = 6 +1 [Carryover] = 7, put to left)

Step. 8 : 147368421 (as in the same process)

Step. 9 : 947368421 (Do – continue to step 18)

Step. 10 : 18947368421

Step. 11 : 178947368421

Step. 12 : 1578947368421

Step. 13 : 11578947368421

Step. 14 : 31578947368421

Step. 15 : 631578947368421

Step. 16 : 12631578947368421

Step. 17 : 52631578947368421

Step. 18 : 1052631578947368421

Now from step 18 onwards the same numbers and order towards left continue.

Thus 1 / 19 = 0.052631578947368421

b) Division Method : Value of 1 / 19.

The numbers of decimal places before repetition is the difference of numerator and denominator, i.e.,, 19 -1=18 places.
For the denominator 19, the purva (previous) is 1.
Hence Ekadhikena purva (one more than the previous) is 1 + 1 = 2.
The sutra is applied in a different context. Now the method of division is as follows:

Step. 1 : Divide numerator 1 by 20.
i.e.,, 1 / 20 = 0.1 / 2 = .10 (0 times, 1 remainder)

Step. 2 : Divide 10 by 2

i.e.,, 0.005(5 times, 0 remainder)

Step. 3 : Divide 5 by 2

i.e.,, 0.0512 (2 times, 1 remainder)

Step. 4 : Divide 12 i.e.,, 12 by 2

i.e.,, 0.0526 (6 times, No remainder)

Step. 5 : Divide 6 by 2

i.e.,, 0.05263 (3 times, No remainder)

Step. 6 : Divide 3 by 2

i.e.,, 0.0526311(1 time, 1 remainder)

Step. 7 : Divide 11 i.e.,, 11 by 2

i.e.,, 0.05263115 (5 times, 1 remainder)

Step. 8 : Divide 15 i.e.,, 15 by 2

i.e.,, 0.052631517 (7 times, 1 remainder)

Step. 9 : Divide 17 i.e.,, 17 by 2

i.e.,, 0.05263157 18 (8 times, 1 remainder)

Step. 10 : Divide 18 i.e.,, 18 by 2

i.e.,, 0.0526315789 (9 times, No remainder)

Step. 11 : Divide 9 by 2

i.e.,, 0.0526315789 14 (4 times, 1 remainder)

Step. 12 : Divide 14 i.e.,, 14 by 2

i.e.,, 0.052631578947 (7 times, No remainder)

Step. 13 : Divide 7 by 2

i.e.,, 0.05263157894713 (3 times, 1 remainder)

Step. 14 : Divide 13 i.e.,, 13 by 2

i.e.,, 0.052631578947316 (6 times, 1 remainder)

Step. 15 : Divide 16 i.e.,, 16 by 2

i.e.,, 0.052631578947368 (8 times, No remainder)

Step. 16 : Divide 8 by 2

i.e.,, 0.0526315789473684 (4 times, No remainder)

Step. 17 : Divide 4 by 2

i.e.,, 0.05263157894736842 (2 times, No remainder)

Step. 18 : Divide 2 by 2

i.e.,, 0.052631578947368421 (1 time, No remainder)

Now from step 19, i.e.,, dividing 1 by 2, Step 2 to Step. 18 repeats thus giving

1 / 19 =0.052631578947368421 or 0.052631578947368421

CLASS VII & VIII - Exercise 3

Find the recurring decimal form of the fractions 1 / 29, 1 / 59, 1 / 69, 1 / 79, 1 / 89 using Ekadhika process

SUTRA 2 - *Nikhilam navatascaramam Dasatah*

The formula simply means **: "all from 9 and the last from 10"**

The formula can be very effectively applied in multiplication of numbers, which are nearer to bases like 10, 100, 1000i.e., to the powers of 10 . The procedure of multiplication using the Nikhilam involves minimum number of steps, space, time saving and only mental calculation. The numbers taken can be either less or more than the base considered.

The difference between the number and the base is termed as deviation. Deviation may be positive or negative. Positive deviation is written without the positive sign and the negative deviation, is written using Rekhank (a bar on the number). Now observe the following table.

Number	Base	Number – Base	Deviation
14	10	14 - 10	4
8	10	8 - 10	-2 or $\overline{2}$
97	100	97 - 100	-03 or $\overline{03}$
112	100	112 - 100	12
993	1000	993 - 1000	-007 or $\overline{007}$
1011	1000	1011 - 1000	011

Eg : Multiply 7 by 8.

The R.H.S. of the answer is the product of the deviations of the numbers. It shall contain the number of digits equal to number of zeroes in the base.

_
i.e., 7 3

_
8 2

/ (3x2) = 6

Since base is 10, 6 can be taken as it is.

e) L.H.S of the answer is the sum of one number with the deviation of the other. It can be arrived at in any one of the four ways.

i) Cross-subtract deviation 2 on the second row from the original number7 in the first row i.e., 7-2 = 5.

ii) Cross–subtract deviation 3 on the first row from the original number8 in the second row (converse way of(i)) i.e., 8 - 3 = 5

iii) Subtract the base 10 from the sum of the given numbers.., (7 + 8) – 10 = 5

iv) Subtract the sum of the two deviations from the base i.e., 10 – (3 + 2) = 5

Hence 5 is left hand side of the answer.

_Thus 7 3
_ 8 2

5 /

Now (d) and (e) together give the solution

_7 3 7
_8 2 i.e., X 8

5 / 6 56

f) If R.H.S. contains less number of digits than the number of zeros in the base, the remaining digits are filled up by giving zero or zeroes on the left side of the R.H.S. If the number of digits are more than the number of zeroes in the base, the excess digit or digits are to be added to L.H.S of the answer.

Ex. 1: Find 97 X 94.

Here base is 100. Now following the rules, the working is as follows:

97 $\overline{03}$

94 $\overline{06}$

(97 - 06) or (94 - 03) / 3 X 6

Ex. 2: 986 X 989. Base is 1000.

986 $\overline{014}$

989 $\overline{011}$

(986 - 14) or (989 - 14) / 14 X 11 = 975/154 = 975154

.

Case (ii) : Both the numbers are higher than the base.
The method and rules follow as they are. The only difference is the positive deviation. Instead of cross – subtract, we follow cross – add.

Ex. 3: 13X12. Base is 10

13 03

12 02

(13 + 02) or (12 + 03) / 3 X 2 = 15/6 = 156

Ex. 4: 104X102. Base is 100.

104 04

102 02

106 / 4x2 = 10608 (rule -f)

Ex. 5: 1275X1004. Base is 1000.

1275	275
1004	004

1279/ 275x4 = 1279 / 1100 (rule -f)

_____________ = 1280100

Case (iii): One number is more and the other is less than the base.

In this situation one deviation is positive and the other is negative. So the product of deviations becomes negative. So the right hand side of the answer obtained will therefore have to be subtracted. To have a clear representation and understanding a vinculum is used. It proceeds into normalization.

Ex. 6 13X7. Base is 10

13	03
7	$\overline{03}$

(13 - 03) or (7 + 03) / $3 \times \bar{3}$ = 10 / $\bar{9}$ = 100-9 = 91

Ex. 7: 108 X 94. Base is 100.

108	08
94	$\overline{06}$

(108 - 06) or (94 + 08) / $8 \times \bar{6}$ = 102 / $\overline{48}$ = 10152

(Since the complement of 48 is 52 - base 10)

Ex. 8: 998 X 1025. Base is 1000.

998	002
1025	$\overline{025}$

(998 - 25) or (1025 + 2) / $\bar{2} \times 25$ = 1023 / $\overline{050}$ = 1022950

(Since the complement of 50 is 950 for the base 1000

CLASS VI , VII & VIII - Exercise 4

Find the following products by Nikhilam formula.

1) 7 X 4 2) 93 X 85 3) 875 X 994 4) 1234 X 1002 5) 1003 X 997 6) 11112 X 9998 7) 1234 X 1002 8) 118 X 105

Nikhilam in Division

Consider some two digit numbers (dividends) and same divisor 9. Observe the following example.

i) 13 ÷ 9 The quotient (Q) is 1, Remainder (R) is 4.

ii) 34 ÷ 9, Q is 3, R is 7.

iii) 60 ÷ 9, Q is 6, R is 6.

iv) 80 ÷ 9, Q is 8, R is 8.

Now we have another type of representation for the above examples as given hereunder:

i) Split each dividend into a left hand part for the Quotient and right - hand part for the remainder by a slant line or slash.

Eg. 13 as 1 / 3, 34 as 3 / 4 , 80 as 8 / 0.

ii) Leave some space below such representation, draw a horizontal line.

Now we have another type of representation for the above examples as given hereunder:

i) Split each dividend into a left hand part for the Quotient and right - hand part for the remainder by a slant line or slash.

Eg. 13 as 1 / 3, 34 as 3 / 4 , 80 as 8 / 0.

ii) Leave some space below such representation, draw a horizontal line.

Eg. 1 / 3   3 / 4   8 / 0

____ ,   ______ ,   ____

iii) Put the first digit of the dividend as it is under the horizontal line. Put the same digit under the right hand part for the remainder, add the two and place the sumi.e., sum of the digits of the numbers as the remainder.

Eg.

1 / 3	3 / 4	8 / 0
1	3	8
______ ,	______ ,	______
1 / 4	3 / 7	8 / 8

Now in the case of 3 digit numbers, let us proceed as follows.

```
ii)
            9 ) 212 ( 23              9 ) 21 / 2
                207                        2 / 3
               -----                   -----------
                                as
                 5                      23 / 5

iii)
            9 ) 401 (44              9 ) 40 / 1
                396                        4 / 4
               -----                  -----------
                               as
                 5                      44 / 5
```

Consider 511 ÷ 9

Add the first digit 5 to second digit 1 getting 5 + 1 = 6. Hence Quotient is 56. Now second digit of 56 i.e., 6 is added to third digit 1 of dividend to get the remainder i.e., 1 + 6 = 7

Eg : 1204 ÷ 9

i) Add first digit 1 to the second digit 2. 1 + 2 = 3
ii) Add the second digit of quotient 13. i.e., 3 to third digit ‘0’ and obtain the Quotient. 3 + 0 = 3, 133
iii) Add the third digit of Quotient 133 i.e.,3 to last digit ‘4’ of the dividend and write the final Quotient and Remainder. R = 3 + 4 = 7, Q = 133

CLASS VI , VII , & VIII - Exercise 5

Obtain the Quotient and Remainder for the following problems.

1) 311 ÷ 9   2) 120012 ÷ 9   3) 1135 ÷ 97 4) 2342 ÷ 98

5) 113401 ÷ 997   6) 11199171 ÷ 99979

SUTRA 3 - *Urdhva – tiryagbhyam*

Urdhva – tiryagbhyam is the general formula applicable to all cases of multiplication and also in the division of a large number by another large number.

Ex.1: Find the product 14 X 12

i) The right hand most digit of the multiplicand, the first number (14) i.e.,4 is multiplied by the right hand most digit of the multiplier, the second number (12)i.e., 2. The product 4 X 2 = 8 forms the right hand most part of the answer.

ii) Now, diagonally multiply the first digit of the multiplicand (14) i.e., 4 and second digit of the multiplier (12)i.e., 1 (answer 4 X 1=4); then multiply the second digit of the multiplicand i.e.,1 and first digit of the multiplier i.e., 2 (answer 1 X 2 = 2); add these two i.e.,4 + 2 = 6. It gives the next, i.e., second digit of the answer. Hence second digit of the answer is 6.

iii) Now, multiply the second digit of the multiplicand i.e., 1 and second digit of the multiplieri.e., 1 vertically, i.e., 1 X 1 = 1. It gives the left hand most part of the answer.

Thus the answer is 16 8.

Ex.2 28 X 35.

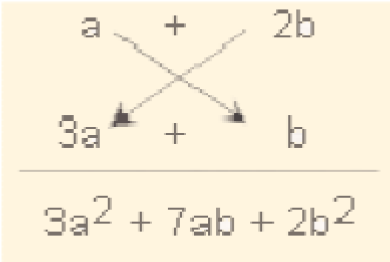
Step (i) : 8 X 5 = 40. 0 is retained as the first digit of the answer and 4 is carried over.

Step (ii) : 2 X 5 = 10; 8 X 3 = 24; 10 + 24 = 34; add the carried over 4 to 34. Now the result is 34 + 4 = 38. Now 8 is retained as the second digit of the answer and3 is carried over.

Step (iii) : 2 X 3 = 6; add the carried over 3 to 6. The result 6 + 3 = 9 is the third or final digit from right to left of the answer.

Thus 28 X 35 = 980.

Example 1 : Find the product of (a+2b) and (3a+b).



Example 3 : $(3a^2 + 2a + 4) \times (2a^2 + 5a + 3)$

i) 4 X 3 = 12

ii) (2 X 3) + (4 X 5) = 6 + 20 =26 i.e., 26a

iii) (3 X 3) + (2 X 5) + (4 X 2) = 9 + 10 + 8 = 27 i.e., $27a^2$

iv) (3 X 5) + (2 X 2) = 15 + 4 = 19 i.e., 19 a^3

v) 3 X 2 = 6 i.e.,$6a^4$

Hence the product is $6a^4 + 19a^3 + 27a^2 + 26a + 12$

CLASS VII & VIII Exercise 6

Find the products using urdhva tiryagbhyam process.

1) 25 X 16 2) 32 X 48 3) 56 X 56 4) 137 X 214 5) 321 X 213

6) 452 X 348 7) (2x + 3y) (4x + 5y) 8) $(5a^2 + 1)$ (3a2 + 4)

9) $(6x^2 + 5x + 2)(3x^2 + 4x + 7)$ 10) $(4x^2 + 3)(5x + 6)$

SUTRA 4 - *Paravartya Yojayet*

'Paravartya – Yojayet' means 'transpose and apply'

(i) Consider the division by divisors of more than one digit, and when the divisors are slightly greater than powers of 10.B B

Example 1 : Divide 1225 by 12.

Step 1 : (From left to right) write the Divisor leaving the first digit, write the other digit or digits using negative (-) sign and place them below the divisor as shown.

12
-2

Step 2 : Write down the dividend to the right. Set apart the last digit for the remainder.

i.e.,, 12 122 5
- 2

Step 3 : Write the 1st digit below the horizontal line drawn under the dividend. Multiply the digit by –2, write the product below the 2nd digit and add.

i.e.,, 12 122 5
-2 -2

10

Since 1 x –2 = -2and 2 + (-2) = 0

Step 4 : We get second digits' sum as '0'. Multiply the second digits' sum thus obtained by –2 and writes the product under 3rd digit and add.

12 122 5
- 2 -20

102 5

Step 5 : Continue the process to the last digit.

i.e., 12 122 5
- 2 -20 -4

102 1

Step 6: The sum of the last digit is the Remainder and the result to its left is Quotient. **Thus Q = 102 and R = 1**

CLASS VII & VIII – Exercise 7

Find the Quotient and Remainder for the problems using paravartya – yojayet method.

1) 1234 ÷ 112 2) 11329 ÷ 1132 3) 12349÷ 133 4) 239479÷1203

SUTRA 5 - Sunyam Samya Samuccaye

The Sutra **'Sunyam Samyasamuccaye'** says the 'Samuccaya is the same, that Samuccaya is Zero.' i.e., it should be equated to zero. The term 'Samuccaya' has several meanings under different contexts.

i) We interpret, 'Samuccaya' as a term which occurs as a common factor in all the terms concerned and proceed as follows.

Example 1: The equation $7x + 3x = 4x + 5x$ has the same factor ' x ' in all its terms. Hence by the sutra it is zero,i.e., $x = 0$.

Otherwise we have to work like this:

$7x + 3x = 4x + 5x$

$10x = 9x$

$10x – 9x = 0$

$x = 0$

Example 2: 5(x+1) = 3(x+1)

No need to proceed in the usual procedure like

$5x + 5 = 3x + 3$

$5x – 3x = 3 – 5$

$2x = -2$ or $x = -2 \div 2 = -1$

Simply think of the contextual meaning of 'Samuccaya'

Now Samuccaya is $(x + 1)$

$x + 1 = 0$ gives $x = -1$

Example 3: $(x + 3) (x + 4) = (x – 2) (x – 6)$

Here Samuccaya is $3 \times 4 = 12 = -2 \times -6$ Since it is same , we derive $x = 0$

Example 4:

$$\frac{3x+4}{3x+5} = \frac{3x+5}{3x+4}$$

Since N1 + N2 = 3x + 4 + 3x + 5 = 6x + 9 ,

And D1 + D2 = 3x + 4 + 3x + 5 = 6x + 9

We haveN1 + N2 = D1 + D2 = 6x + 9

Hence from Sunya Samuccaya we get 6x + 9 = 0   6x = -9

CLASS VIII - Exercise - 8

Solve the following problems using Sunyam Samya-Samuccaye process.

1. 7 (x + 2) + 3 (x + 2) = 6 (x + 2) + 5 (x + 2)

2. (x + 6) (x + 3) = (x – 9) (x – 2)

3. (x - 1) (x + 14) = (x + 2) (x – 7)

4. $$\frac{1}{4x-3} + \frac{1}{x-2} = 0$$

5. $$\frac{4}{3x+1} + \frac{4}{5x+7} = 0$$

6. $$\frac{2x+11}{2x+5} = \frac{2x+5}{2x+11}$$

7. $$\frac{3x+4}{6x+7} = \frac{x+1}{2x+3}$$

SUTRA 6 - Anurupye - Sunyamanyat

The **Sutra Anurupye Sunyamanyat says** : 'If one is in ratio, the other one is zero'. We use this Sutra in solving a special type of simultaneous simple equations in which the coefficients of 'one' variable are in the same ratio to each other as the independent terms are to each other. In such a context the Sutra says the 'other' variable is zero from which we get two simple equations in the first variable (already considered) and of course give the same value for the variable.

Example 1: **solve** $3x + 7y = 2$ $4x + 21y = 6$

Observe that the y-coefficients are in the ratio 7 : 21 i.e., 1 : 3, which is same as the ratio of independent terms i.e., 2 : 6 i.e., 1 : 3. Hence the other variable $x = 0$ and $7y = 2$ or $21y = 6$ gives $y = 2 / 7$

Example 2: Solve $323x + 147y = 1615$: $969x + 321y = 4845$

The very appearance of the problem is frightening. But just an observation and anurupye sunyamanyat give the solution $x = 5$, because coefficient of x ratio is 323 : 969 = 1 : 3 and constant terms ratio is 1615 : 4845 = 1 : 3.

$y = 0$ and $323x = 1615$ or $969x = 4845$ gives $x = 5$.

CLASS VIII - Exercise - 9

Solve the following by anurupye sunyamanyat.

$4x – 6y = 24$ $12x + 78y = 12$ $3x + 7y = 24$

16x + 96y =16	12x + 5y = 96	7x – 9y = 36

SUTRA 7 - Sankalana – Vyavakalanabhyam

This Sutra means 'by addition and by subtraction'. It can be applied in solving a special type of simultaneous equations where the x - coefficients and the y - coefficients are found interchanged.

Example 1:

45x – 23y = 113
23x – 45y = 91

From **Sankalana – vyavakalanabhyam**

add them,

i.e., (45x – 23y) + (23x – 45y) = 113 + 91
i.e., 68x – 68y = 204 x – y = 3

subtract one from other,

i.e., (45x – 23y) – (23x – 45y) = 113 – 91
i.e., 22x + 22y = 22 x + y = 1
and repeat the same sutra, we get x = 2 and y = - 1
Very simple addition and subtraction are enough, however big the coefficients may be.

Example 2:

1955x – 476y = 2482
476x – 1955y = -4913
Oh ! what a problem ! And still
just add, 2431(x – y) = - 2431 x – y = -1
subtract, 1479 (x + y) = 7395 x + y = 5
once again add, 2x = 4 x = 2
subtract - 2y = - 6 y = 3

CLASS VIII – Exercise – 10

Solve the following problems usingSankalana – Vyavakalanabhyam.

1. 3x + 2y = 18 2x + 3y = 17
2. 5x – 21y = 26 21x – 5y = 26
3. 659x + 956y = 4186 956x + 659y = 3889

SUTRA 8 - *Puranapuranabhyam*

The Sutra can be taken as **Purana - Apuranabhyam** which means by the completion or non - completion. Purana is well known in the present system. We can see its application in solving the roots for general form of quadratic equation.

Example 1. $x^3 + 6x^2 + 11x + 6 = 0$.
Since $(x + 2)^3 = x^3 + 6x^2 + 12x + 8$ Add $(x + 2)$ to both sides
We get $x^3 + 6x^2 + 11x + 6 + x + 2 = x + 2$
i.e.,, $x^3 + 6x^2 + 12x + 8 = x + 2$
i.e.,, $(x + 2)^3 = (x + 2)$
this is of the form $y^3 = y$ for $y = x + 2$ solution $y = 0$, $y = 1$, $y = -1$
i.e.,, $x + 2 = 0,1,-1$ which gives $x = -2,-1,-3$

Example 2: $x^3 + 8x^2 + 17x + 10 = 0$
We know $(x + 3)^3 = x^3 + 9x^2 + 27x + 27$
So adding on the both sides, the term (x2 + 10x + 17), we get
$x^3 + 8x^2 + 17x + x^2 + 10x + 17 = x^2 + 10x + 17$
i.e.,, $x^3 + 9x^2 + 27x + 27 = x^2 + 6x + 9 + 4x + 8$
i.e.,, $(x + 3)^3 = (x + 3)^2 + 4(x + 3) - 4$
$y^3 = y^2 + 4y - 4$ for $y = x + 3$ $y = 1, 2, -2$. Hence $x = -2, -1, -5$
Thus purana is helpful in factorization.
Further purana can be applied in solving Biquadratic equations also.

CLASS VIII – Exercise – 11

Solve the following using purana – apuranabhyam.

1. $x^3 - 6x^2 + 11x - 6 = 0$
2. $x^3 + 9x^2 + 23x + 15 = 0$
3. $x^2 + 2x - 3 = 0$

4. $x^4 + 4x^3 + 6x^2 + 4x - 15 = 0$

SUTRA 9 - Calana – Kalanabhyam

The Sutra means 'Sequential motion'.
In the first instance it is used to find the roots of a quadratic equation $7x^2 - 11x - 7 = 0$.

Further other Sutras 10 to 16 mentioned below are also used to get the required results. Hence the sutra and its various applications will be taken up at a later stage for discussion.
But sutra – 14 is discussed immediately after this item.

SUTRA 10 - Ekanyunena Purvena

The Sutra **Ekanyunena purvena** comes as a Sub-sutra to Nikhilam which gives the meaning 'One less than the previous' or 'One less than the one before'.
1) The use of this sutra in case of multiplication by 9,99,999.. is as follows .

The left hand side digit (digits) is (are) obtained by applying the ekanyunena purvena i.e. by deduction 1 from the left side digit (digits) .
e.g. (i) 7 x 9; 7 – 1 = 6 (L.H.S. digit)
b) The right hand side digit is the complement or difference between the multiplier and the left hand side digit (digits) . i.e. 7 X 9 R.H.S is 9 - 6 = 3.
c) The two numbers give the answer; i.e. 7 X 9 = 63.

Example 1: 8 x 9 **Step (a)** gives 8 – 1 = 7 (L.H.S. Digit)

Step (b) gives 9 – 7 = 2 (R.H.S. Digit)

Step (c) gives the answer 72

Example 2: 15 x 99 **Step (a) :** 15 – 1 = 14

Step (b) : 99 – 14 = 85 (or 100 – 15)

Step (c) : 15 x 99 = 1485

Example 3: 24 x 99

Answer :

(24 - 1) / (99 - 23)

= 23 / = 76 (or 100 - 24) = 2376

Example 4: 356 x **999**

(356 - 1) / (999 - 335)

= 355 / = 644 = 355644

CLASS VI , VII & VIII Exercise – 12

Apply Ekanyunena purvena to find out the products

1. 64 x 99	2. 723 x 999	3. 3251 x 9999
4. 43 x 999	5. 256 x 9999	6. 1857 x 99999

SUTRA 11 - *Anurupyena*

The **upa-Sutra 'anurupyena' means 'proportionality'.** This Sutra is highly useful to find products of two numbers when both of them are near the Common bases i.e powers of base 10 . It is very clear that in such cases the expected 'Simplicity ' in doing problems is absent.

Example 1: 46 X 43

As per the previous methods, if we select 100 as base we get 46 -54 and 43 - 53 This is much more difficult and of no use.

Now by 'anurupyena' we consider a working base In three ways. We can solve the problem.

Method 1: For the **example 1:** 46X43. We take the same working base 50. We treat it as 50=5X10.

i.e. we operate with 10 but not with 100 as in method

46	-04
43	-07
(46 - 7) or (43 - 4) = 39	-4 x -7 = $_2$8
39 x 5 +2 (carried over)	8

(195 + 2) / 8 = 1978

[Since we operate with 10, the R.H.S portion shall have only unit place .Hence out of the product 28, 2 is carried over to left side. The L.H.S portion of the answer shall be multiplied by 5, since we have taken 50 = 5 X 10.]

Example 5: 3998 X 4998

Working base = 10000 / 2 = 5000

3998	-1002	
4998	-0002	since 2 is carried out
3996	$_2$004	
X 5		
19980	$_2$004	= 19982004

CLASS VI , VII , VIII – Exercise – 13

Use anurupyena by selecting appropriate working base and method. Find the following product.

1. 46 x 46	2. 57 x 57	3. 54 x 45	4. 18 x 18
5. 62 x 48	6. 229 x 230	7. 47 x 96	8. 87965 x 99996
9. 49x499	10. 389 x 512		

SUTRA 12 - *Adyamadyenantya – mantyena*

The Sutra ' **adyamadyenantya-mantyena'** means 'the first by the first and the last by the last'.

Suppose we are asked to find out the area of a rectangular card board whose length and breadth are respectively 6ft . 4 inches and 5 ft. 8 inches. Generally we continue the problem like this.

Area = Length X Breath

= 6’ 4" X 5’ 8" Since 1’ = 12", conversion

= (6 X 12 + 4) (5 X 12 + 8) in to single unit

= 76" 68" = 5168 Sq. inches.

By Vedic principles we proceed in the way "the first by first and the last by last"

i.e. 6’ 4" can be treated as 6x + 4 and 5’ 8" as 5x + 8,

Where x= 1ft. = 12 in;x2 is sq. ft.

Now (6x + 4)(5x + 8)

= 30x2 + 6.8.x + 4.5.x + 32

= 30x2 + 48x + 20x + 32

= 30x2 + 68. x + 32

= 30x2 + (5x + 8). x + 32 Writing 68 = 5 x 12 + 8

= 35x2 + 8. x + 32

= 35 Sq. ft. + 8 x 12 Sq. in + 32 Sq. in

= 35 Sq. ft. + 96 Sq. in + 32 Sq. in

= 35 Sq. ft. + 128 Sq. in

CLASS VIII – Exercise – 14

I. Find the area of the rectangles in each of the following situations.

1). l = 3’ 8" , b = 2’ 4 " 2). l = 12’ 5" , b = 5’ 7"

3). l = 4 yard 3 ft. b = 2 yards 5 ft.(1yard =3ft)

4). l = 6 yard 6 ft. b = 5 yards 2 ft.

SUTRA 13 - *Yavadunam Tavadunikrtya Varganca Yojayet*

The meaning of the Sutra is 'what ever the deficiency subtract that deficit from the number and write along side the square of that deficit'.
This Sutra can be applicable to obtain squares of numbers close to bases of powers of 10.
Method-1 : Numbers near and less than the bases of powers of 10.

Eg 1: 9^2 Here base is 10.

The answer is separated in to two parts by a'/'

Note that deficit is 10 - 9 = 1

Multiply the deficit by itself or square it

12 = 1. As the deficiency is 1, subtract it from the number i.e., 9–1 = 8.

Now put 8 on the left and 1 on the right side of the vertical line or slash

i.e., 8/1.

Eg. 2: 96^2 Here base is 100.

Since deficit is 100-96=4 and square of it is 16 and the deficiency

subtracted from the number 96 gives 96-4 = 92, we get the answer 92 / 16

Thus 96^2 = 9216.

Eg. 3: 994^2 Base is 1000

Deficit is 1000 - 994 = 6. Square of it is 36.

Deficiency subtracted from 994 gives 994 - 6 = 988

Answer is 988 / 036 [since base is 1000]

Eg. 4: 9988^2 Base is 10,000.

Deficit = 10000 - 9988 = 12.

Square of deficit = 122 = 144.

Deficiency subtracted from number = 9988 - 12 = 9976.

Answer is 9976 / 0144 [since base is 10,000].

Eg. 5: 88^2 Base is 100.

Deficit = 100 - 88 = 12.

Square of deficit = 122 = 144.

Deficiency subtracted from number =88 - 12 = 76.

Now answer is 76 / 144 =7744 [since base is 100]

Method. 2 : Numbers near and greater than the bases of powers of 10.

Eg.(1): 13^2 .

Instead of subtracting the deficiency from the number we add and proceed as in Method-1.

for 132 , base is 10, surplus is 3.

Surplus added to the number = 13 + 3 = 16.

Square of surplus = 32 = 9

Answer is 16 / 9 = 169.

Eg.(2): 112^2

Base = 100, Surplus = 12,

Square of surplus = 122 = 144

add surplus to number = 112 + 12 = 124.

Answer is 124 / 144 = 12544

CLASS VI , VII & VIII – Exercise - 15

Apply yavadunam to find the following squares.

1. 7^2	2. 98^2	3. 987^2	4. 14^2
5. 116^2	6. 1012^2	7. 192	8. 475^2
9. 796^2	10. 108^2	11. 9988^2	12. 6014^2.

Cubing of Numbers:

Example : Find the cube of the number 106.

We proceed as follows:

i) For 106, Base is 100. The surplus is 6.
Here we add double of the surplus i.e. 106+12 = 118.
(Recall in squaring, we directly add the surplus)
This makes the left-hand -most part of the answer.
i.e. answer proceeds like 118 / - - - - -

ii) Put down the new surplus i.e. 118-100=18 multiplied by the initial surplus i.e. 6=108.
Since base is 100, we write 108 in carried over form 108 i.e. .
As this is middle portion of the answer, the answer proceeds like 118 / 108 /....

iii) Write down the cube of initial surplus i.e. 63 = 216 as the last portion i.e. right hand side last portion of the answer.
Since base is 100, write 216 as 216 as 2 is to be carried over.
Answer is 118 / 108 / 216
Now proceeding from right to left and adjusting the carried over, we get the answer 119 / 10 / 16 = 1191016.

Eg.(1): 1023 = (102 + 4) / 6 X 2 / 23
= 106 = 12 = 08
= 1061208.
Observe initial surplus = 2, next surplus =6 and base = 100.

Eg.(2): 94^3
Observe that the nearest base = 100. Here it is deficit contrary to the above examples.

i) Deficit = -6. Twice of it -6 X 2 = -12
add it to the number = 94 -12 =82.
ii) New deficit is -18.
Product of new deficit x initial deficit = -18 x -6 = 108
iii) deficit3 = (-6)3 = -216.
__
Hence the answer is 82 / 108 / -216
Since 100 is base 1 and -2 are the carried over. Adjusting the carried over in order, we get the answer
(82 + 1) /(08 – 03) / (100 – 16)
= 83 / = 05 / = 84 = 830584
16 becomes 84 after taking1 from middle most portion i.e. 100. (100-16=84). Now 08 - 01 = 07 remains in the middle portion, and2 or 2 carried to it makes the middle as 07 - 02 = 05. Thus we get the above result.

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Now 08 - 01 = 07 remains in the middle portion, and2 or 2 carried to it makes the middle as 07 - 02 = 05. Thus we get the above result.

Eg.(3): 998^3 Base = 1000; initial deficit = - 2.

9983 = (998 – 2 x 2) / (- 6 x – 2) / (- 2)3

= 994 / = 012 / = -008

= 994 / 011 / 1000 - 008

= 994 / 011 / 992

= 994011992.

CLASS VII & VIII – Exercise – 16

Find the cubes of the following numbers using yavadunam sutra.

1. 105	2. 114	3. 1003	4. 10007	5. 92
6. 96	7. 993	8. 9991	9. 1000008	10. 999992.

SUTRA 14 - *Antyayor Dasakepi*

The Sutra signifies numbers of which the last digits added up give 10.

i.e. the Sutra works in multiplication of numbers for example: 25 and 25, 47 and 43, 62 and 68, 116 and 114. Note that in each case the sum of the last digit of first number to the last digit of second number is 10. Further the portion of digits or numbers left wards to the last digits remain the same. At that instant use Ekadhikena on left hand side digits. Multiplication of the last digits gives the right hand part of the answer.

Example 1 : 47 X 43

See the end digits sum 7 + 3 = 10 ; then by the sutras antyayor dasakepi and ekadhikena we have the answer.

47 x 43 = (4 + 1) x 4/ 7 x 3

= 20 / 21

= 2021.

Example 2: 62 x 68

2 + 8 = 10, L.H.S. portion remains the same i.e.,, 6.

Ekadhikena of 6 gives 7

62 x 68 = (6 x 7)/ (2 x 8)

= 42 / 16

= 4216.

Example 3: 127 x 123

As antyayor dasakepi works, we apply ekadhikena

127 x 123 = 12 x 13/ 7 x 3

= 156 / 21

= 15621.

Example 4: 395^2

3952 = 395 x 395

= 39 x 40 / 5 x 5

= 1560 / 25

= 156025.

CLASS VI , VII , VIII – Exercise – 17

Use Vedic sutras to find the products

1. 125 x 125	2. 34 x 36	3. 98 x 92
4. 401 x 409	5. 693 x 697	6. 1404 x 1406

Eg. 5: 292 x 208

Here 92 + 08 = 100, L.H.S portion is same i.e. 2

292 x 208 = (2 x 3)/ 92 x 8

60 / =736 (for 100 raise the L.H.S. product by 0)

= 60736.

Eg. 6: 693 x 607

693 x 607 = 6 x 7/ 93 x 7

= 420 / 651

= 420651.

CLASS VII & VIII – Exercise - 18

Find the following products using ‘antyayordasakepi’

1. 318 x 312 2. 425 x 475 3. 796 x 744

4. 902 x 998	5. 397 x 393	6. 551 x 549

SUTRA 15 - *Antyayoreva*

'Atyayoreva' means 'only the last terms'. This is useful in solving simple equations of the following type.

The type of equations are those whose numerator and denominator on the L.H.S. bearing the independent terms stand in the same ratio to each other as the entire numerator and the entire denominator of the R.H.S. stand to each other.

SUTRA 16 - *Lopana Sthapanabhyam*

To find the Highest Common Factor i.e. H.C.F. of algebraic expressions, the factorization method and process of continuous division are in practice in the conventional system. We now apply' Lopana - Sthapana' Sutra, the 'Sankalana vyavakalanakam' process and the 'Adyamadya' rule to find out the H.C.F in a more easy and elegant way.

Example 1: Find the H.C.F. of $x^2 + 5x + 4$ and $x^2 + 7x + 6$.

1. Factorization method:

$x^2 + 5x + 4 = (x + 4)(x + 1)$
$x^2 + 7x + 6 = (x + 6)(x + 1)$
H.C.F. is $(x + 1)$.

Example 2: Find H.C.F. of $2x^2 - x - 3$ and $2x^2 + x - 6$

Subtract
$$\begin{array}{r} 2x^2 - x - 3 \\ 2x^2 + x - 6 \\ \hline -1)\ -2x + 3 \\ \hline 2x + 3 \text{ is the H.C.F} \end{array}$$

Example 4: $x^3 + 6x^2 + 5x - 12$ and $x^3 + 8x^2 + 19x + 12$.

Add, $\div 2x$
$$\begin{array}{r} x^3 + 6x^2 + 5x - 12 \\ x^3 + 8x^2 + 19x + 12 \\ \hline 2x^3 + 14x^2 + 24x \\ \hline x^2 + 7x + 12 \end{array}$$

(or)

Subtract, $\div -2$
$$\begin{array}{r} x^3 + 6x^2 + 5x - 12 \\ x^3 + 8x^2 + 19x + 12 \\ \hline 2x^2 - 14x - 24 \\ \hline x^2 + 7x + 12 \end{array}$$

CLASS VIII - Exercise - 19

Find the H.C.F. in each of the following cases using Vedic sutras:

1. $x^2 + 2x - 8$, $x^2 - 6x + 8$
2. $x^3 - 3x^2 - 4x + 12$,$x^3 - 7x^2 + 16x - 12$
3. $x^3 + 6x^2 + 11x + 6$, $x^3 - x^2 - 10x - 8$
4. $6x4 - 11x^3 + 16x^2 - 22x + 8$, $6x^4 - 11x^3 - 8x^2 + 22x - 8$.