INDIAN SCHOOL SOHAR **SET II**

PRE-BOARD EXAMINATION -I (2023-24)

MATHEMATICS (CODE- 041)

CLASS: XII **MAX. MARKS: 80**

DATE: 29/11/23 **TIME: 3 Hours**

General Instructions:

1. This question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

	SECTION – A **Multiple Choice Questions** **(Each question carries 1 mark)**	
1.	The function $f: R \rightarrow R$ defined by $\mathrm{f(x)} = 4 + 3\cos x\ is$: (a) bijective (b) one-one but not onto (c) many one and onto (d) neither one one nor onto	**MARKS** 1
2.	If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then $(A-2I)(A-3I)$ is equal to (a) A (b) I (c) 5I (d) O	1
3.	If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ then A^{1001} is equal to (a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1001 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1001 & 0 \\ 0 & 1001 \end{bmatrix}$	1
4.	Given that $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = 3I$ then (a) $1+\alpha^2+\beta\gamma = 0$ (b) $1-\alpha^2-\beta\gamma = 0$ (c) $3+\alpha^2+\beta\gamma = 0$ (d) $3-\alpha^2-\beta\gamma = 0$	1
5.	The inverse of $\begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$ is: (a) $\begin{bmatrix} -5 & -3 \\ 7 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} -5 & 7 \\ 3 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & -7 \\ -3 & 4 \end{bmatrix}$	1
6.	If $\|A\| = 2$ where A is a 2 x 2 matrix, then $\|4A^{-1}\|$ equals to (a) 4 (b) 2 (c) 8 (d) 32	1
7.	The function f(x) =[x], where [x] denotes the greatest integer function, is continuous at (a) 4 (b) -2 (c) 1 d) 1.9	1
8.	If $y = \sin^{-1} x$, then $(1-x^2)y_2$ is equal to (a) xy_1 (b) xy (c) xy_2 (d) x^2	1
9.	If $\frac{d}{dx}\big(f(x)\big) = \log x$, then *f(x)* equals to: (a) $\frac{1}{x} + C$ (b) $-\frac{1}{x} + C$ (c) x(log x +x) + C (d) x(log x – 1)+C	1

10.	The degree of the differential equation $x^2\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^3$ is (a) 1 (b) 2 (c) 6 (d) 3	1
11.	Integrating factor of the differential equation $(1+y^2)\frac{dx}{dy}+yx=ay, (-1<y<1) is$ (a) $\frac{1}{y^2-1}$ (b) $\frac{1}{\sqrt{y^2-1}}$ (c) $\frac{1}{1-y^2}$ (d) $\frac{1}{\sqrt{1-y^2}}$	1
12.	The value of ‘p’ for which the vectors $2\hat{\imath}+p\hat{\jmath}+\hat{k}$ and -4$\hat{\imath}-6\hat{\jmath}+26\hat{k}$ are perpendicular to each other, is (a) 3 (b) -3 (c) $\frac{-17}{3}$ (d) $\frac{17}{3}$	1
13.	$\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is (a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) $\frac{5\pi}{2}$	1
14.	The objective function $z=ax+by$ of an LPP has maximum value at (4, 6) and minimum value 19 at (3, 2). Which of the following is true? (a) a = 9, b = 1 (b) a = 9, b = 2 (c) a = 3, b = 5 (d) a = 5, b = 3	1
15.	If a line makes angles of $90°$, $135°$ and $45°$ with the x, y and z axes respectively, then its direction cosines are (a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$ (c) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$	1
16.	The optimal value of the objective functions is attained at the points (a) given by intersection of in-equation with y-axis (b) given by intersection of in-equation with x-axis (c) given by corner points of the feasible region (d) none of these	1
17.	The number of vectors of unit length perpendicular to the vectors $\vec{a}=2\hat{\imath}+\hat{\jmath}+2\hat{k}$ and $\vec{b}=\hat{\jmath}+\hat{k}$ is (a) one (b) two (c) three (d) infinite	1
18.	A and B are events such that P(A) = 0.4, P(B) = 0.3 and P(B’ $\cap A$) equals (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{10}$ (d) $\frac{1}{5}$	1
	Q. 19 and q. 20 based on Assertion and reason based. **Select the correct answer from the codes (a), (b), (c) and (d) as given below** **(a) Both A and R are true and R is the correct explanation of A** **(b) Both A and R are true and but R is not the correct explanation of A** **(c) A is true and R is false. (d) A is false and R is true.**	1
19.	**Assertion (A) :** If $y=(\sin x+\cos x)^2$, then $\left(\frac{dy}{dx}\right)_{at\ x=\frac{\pi}{4}}=0.$ **Reason (R) :** $\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)$.	1
20.	**Assertion (A):** If $(\vec{a}-\overrightarrow{b}).(\vec{a}+\vec{b})$= 0, then $\vec{a}$ and $\vec{b}$ are perpendicular. **Reason (R):** The projection of $\hat{\imath}+3\hat{\jmath}+\hat{k}$ on $2\hat{\imath}-3\hat{\jmath}+6\hat{k}$ is $\frac{-1}{7}$.	1

SECTION B **(This section comprises of very short answer type questions (VSA) of 2 marks each)**		
21.	Show that : $\tan(\frac{1}{2}\sin^{-1}\frac{3}{4}) = \frac{4-\sqrt{7}}{3}$. **OR** Prove that : $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2}$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	2
22.	Find the absolute maximum and absolute minimum values of $x + \sin 2x$ on $[0, \pi]$.	2
23.	If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.	2
24.	Evaluate : $\int \frac{e^x}{\sqrt{5-4e^x-2e^{2x}}}dx$	2
25.	Find the intervals in which the function $f(x) = 4x^3 - 6x^2 - 72x + 30$ is (a) strictly increasing (b) strictly decreasing. **OR** A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x-coordinate 1, y-coordiate is changing twice as fast at x-coordinate. Find the value of a.	2
SECTION C **(This section comprises of short answer type questions (SA) of 3 marks each)**		
26.	If $f(x) = \begin{cases} \frac{\sin(a+1)x+2\sin x}{x} & ,x < 0 \\ 2 & ,x = \\ \frac{\sqrt{1+bx}-1}{x} & ,x > 0 \end{cases}$ is continuous at x=0, then find the values of a and b. **OR** Differentiate the function with respect to x: $y = (\sin x)^x + \sin^{-1}\sqrt{x}$	3
27.	Evaluate : $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)}dx$	3
28.	Evaluate : $\int_0^{\pi} \frac{x\tan x}{sex\, x + \tan x}dx$ **OR** Evaluate : $\int_0^{\pi/4} \frac{\sin x + \cos x}{16+9\sin 2x}dx$	3
29.	Solve the differential equation $(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$, where $x \in (-\infty, -1) \cup (1, \infty)$. **OR** Solve the differential equation : $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$	3
30.	Solve graphically the following linear programming problem: Maximize: $Z = 6x + 3y$, subject to the contraints $4x + y \geq 80,\ 3x + 2y \leq 150,\ x + 5y \geq 115,\ x, y \geq 0$	3
31.	A coin is biased so that the head is three times likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence find the mean of the tails.	3

SECTION D

(This section comprises of long answer type questions (LA) of 5 marks each)

32. Using the method of integration, find the area of the region bounded by the lines
3x – 2y + 1 = 0, 2x + 3y – 21 = 0 and x – 5y + 9 = 0. **5**

33. Use product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ to solve the system of equations: **5**
$x - y + z = 4, \quad x - 2y - 2z = 9, \quad 2x + y + 3z = 1$

34. Determine whether the relation R defined on the set R of all real numbers as **5**
$R = \{(a, b): a, b \in R \text{ and } a - b + \sqrt{3} \in S, \text{where S is the set of all irrational numbers}\}$,
is reflexive, symmetric and transitive.

OR

A function $f: [-6, 6] \to [0, 6]$ is given by $f(x) = \sqrt{36 - x^2}$. Show that f is an onto function but not one-one function. Further, find all possible values of ‘a’ for which $f(x) = \sqrt{11}$.

35. Find the vector and Cartesian equations of the line which is perpendicular to the lines with the equations $\frac{x+2}{1} = \frac{3-y}{-2} = \frac{z+1}{4}$ and $\frac{1-x}{-2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1). Also find the angle between the given lines. **5**

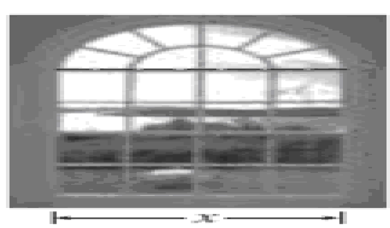
OR

Find the coordinates of the image of the point (1, 6, 3) with respect to the line $\vec{r} = (\hat{j} + 2\hat{k}) - \lambda(-\hat{i} - 2\hat{j} - 3\hat{k})$; where λ scalar. Also, find the distance of the image from the Y-axis.

SECTION E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

36. Amit a student of class XII, visited to his uncle’s house. He observed that the window of the house is in the form of a rectangle surmounted by semicircular opening having perimeter 10m as shown in the figure. Length of the window is x m and width of the window is y m. **4**



Based on the above information, answer the following questions:

(i) The relation between x and y can be represented as

(a) $x + y + \frac{\pi}{2} = 10$ (b) $x + 2y + \frac{\pi x}{2} = 10$

(c) $2x + 2y + \pi x = 10$ (d) $x + 2y + \frac{\pi}{2} = 10$

(ii) Area of the window can be given by

(a) $A = x - \frac{x^2}{8} - \frac{x^3}{2}$ (b) $A = 5x - \frac{x^2}{8} - \frac{\pi x^2}{2}$

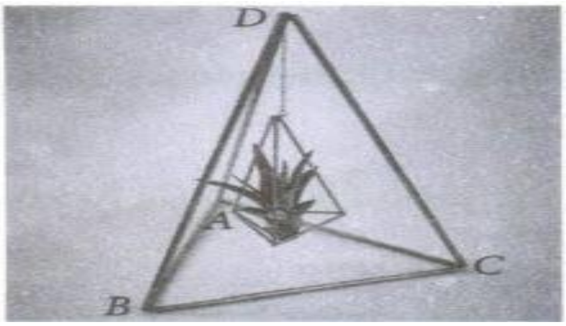
(c) $A = 5x - \frac{\pi x^2}{8} - \frac{x^2}{2}$ (d) $A = x - \frac{\pi x^2}{8} - \frac{3x^2}{2}$

(iii) Amit is interested in maximizing the area of the whole window for this to happen the value of x should be

(a) $\frac{10}{2-\pi}$ (b) $\frac{20}{4-\pi}$ (c) $\frac{20}{4+\pi}$ (d) $\frac{10}{2+\pi}$

(1+1+2)

37. Neha purchased an air plant holder holder which is in the shape of tetrahedron. Let A, B, C and D be the co-ordinate of the air plant holder wher A(1, 2, 3), B(3, 2,1), C(2, 1, 2) and D(3, 4, 3). 4



Based on the above information, answer the following questions: (1+1+2)

(i) Find the vector $\overrightarrow{AB}$

(ii) Find the vector $\overrightarrow{CD}$

(iii) Find the unit vector along vector $\overrightarrow{BC}$

OR

Find the area of ΔBCD

38. Read the following passge and answer the following questions: 4

A shopkeeper sells three types of flower seed A, B, C. They are sold in the form of a mixture, where, the proportions of these seed are 5 : 3 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

(a) Calculate the probability that a randomly chosen seed will germinate.

(b) Calculate the probability that the seed is of type 'B' given that a randomly chosen seed germinates.

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